## DEGREES OF FREEDOM:

## Varying Routes to Math Readiness and the Challenge of Intersegmental Alignment

(Report 2 of a
3-part series)

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## EXECUTIVE SUMMARY

The conventional algebra-intensive math curriculum commonly dictates students' options for entering and completing college, including their ability to transfer from two-year to four-year institutions. The assumption that higher-level algebra is necessary for college success has led some equity advocates to promote algebra for all students. Nearly half of states require two years of algebra for high school graduation, and the Common Core State Standards being implemented in the majority of states have a similar emphasis.

While the intent has been to raise achievement, the hidden underbelly of high algebra expectations has been swelling enrollment in college developmental (also known as remedial) math over the last few decades, especially at community colleges.

After nearly 10 years of investment in improving remedial math success, the initiatives with perhaps the most promise are also the most controversial. Alternative developmental math sequences emphasizing statistics and quantitative reasoning have been developed for students interested in non-algebra-intensive fields. Early
results at community colleges have been extraordinary, opening up opportunities for students to succeed in college. But the sequences defy long-held beliefs, especially within math departments, about the importance of algebra for all students.

Universities' wariness about the initiatives has created a dilemma for students seeking to transfer to universities, leaving colleges hesitant to make alternatives available. The issue is particularly salient in California, given the state's history of experiments with math reforms and its role as the incubator of several alternative math curricula.

## The Transfer Imperative

By design, improving college completion in California requires attention to the transfer route to a bachelor's degree, because the state's master plan limits freshman slots at universities to the top one third of high school graduates, leaving community colleges to ensure access to higher education for the remaining students. Upwards of 60,000 students transfer each year to the two public university systems and nearly 30,000 more to private institutions.

From the universities' perspective, the transfer process works Continued on page 2

EXECUTIVE SUMMARY (Cont.) well. Students who transfer are about as likely to earn bachelor's degrees as juniors who started at four-year universities, in spite of losing some credits in the process. The universities have talked about improving the transfer process, and community colleges face pressure to increase transfer rates. But the universities have little capacity to enroll more transfer students without decreasing freshman enrollment.

## Developmental Math Reforms

Most reforms directed at improving student success in community college address developmental education in some way, because remedial courses represent a major barrier to students' progress, especially in math. The vast majority of community college students are placed into remedial sequences, but recent research reveals that the sequences have insignificant or negative effects on student success. In addition, the standard math sequence is unrelated to most students' career aspirations. Furthermore, assessments misplace a significant proportion of students. These findings have led to innovations in instruction, course placement, and curricular pathways.

Alternative remedial sequences are directed at students who don't major in STEM (Science, Technology, Engineering, and Mathematics) fields, currently about 80 percent of community college grads. Instead of starting with Elementary and Intermediate Algebra and culminating with College Algebra, the alternatives begin with a remedial course emphasizing preparation for Statistics or Quantitative Reasoning and culminate with a college-level Statistics or Quantitative Reasoning course.

Experiments with various alternative versions are taking place at dozens of colleges across the nation, including some state systems. Based on early studies, students in the new sequences are three to four times as successful in passing college-level math requirements as students in standard remedial sequences over a similar or shorter time period.

## California Conundrum: <br> University Limits on Transferability of Reforms

Alternative sequences have faced uncertainty about whether students enrolled in them can transfer smoothly to four-year universities. California's public universities have traditionally expected that the undergraduate math courses required for transfer have a prerequisite of

Intermediate Algebra. After initially rejecting a statistics sequence developed by the Carnegie Foundation for the Advancement of Teaching, the University of California's admissions board recently accepted it. At California State University (CSU), which faces its own remedial math challenges, there remains a concern among math faculty that some alternatives provide insufficient algebra grounding. In fact, in April 2015, CSU math department chairs issued a harsh critique of Carnegie's program. For that reason, some of the experiments are taking place outside of math departments.

The challenge around the transfer of alternative math sequences in California is indicative of community colleges' predicament: As key transit points for students moving through the higher education system, the colleges face an array of expectations that are often confusing, ambiguous, or altogether conflicting.

To address the transfer dilemma, most colleges experimenting with alternatives have avoided listing prerequisites besides Intermediate Algebra. Instead, some California colleges offer them via a loophole that allows students to challenge prerequisites. But reliance on this

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# EXECUTIVE SUMMARY (Cont.) 

workaround makes the programs hard to scale.

## Math Readiness and its Elusive Definition

The discussion of alternatives has laid bare considerable ambiguity about existing math requirements. Despite increasing consensus that students need more grounding in statistics and data analysis, many higher education institutions maintain the traditional emphasis on two years of algebra. But the Algebra 2 that most high schools, colleges, and universities collectively view as the minimal culmination of students' math preparation is ill-defined at best. Statewide alignment in math expectations exists more in name than in practice. Alternative courses may be unfairly bearing scrutiny, since some algebra courses also deviate from the standard curriculum.

Both UC and CSU officials have stated that they don't intend to make a habit of reviewing remedial math courses. UC has also signaled a move away from course titles to a focus on the Common Core math content as the prerequisite for college-level math.

Some universities, including the CSU system, are already imple-
menting various alternative approaches. Since 2002, CSU's placement test has focused on the skills needed in general education quantitative courses, not those needed by students pursuing quantitative majors. There is less emphasis on algebraic manipulation-about 40 percent of traditional Algebra 2 content according to one analysisfreeing colleges to cover less algebra in their remedial sequences.

## Seeking Alignment and Transparency

Though there are few levers for promoting alignment and transparency in California, there remains a need for deeper and more pragmatic dialogue about alignment. California was the first state in which every higher education system endorsed the Common Core standards, but their introduction has yet to become a vehicle for seeking alignment or fostering transparency among the education segments.

Some involved in setting readiness standards hope that the new math standards provide a sensible benchmark for math readiness. But even those standards have been the subject of some confusion, as evidenced by the questions surrounding UC's attempt to clarify its prerequisite standard. Both the new math standards and the alternative sequences
are now being implemented, however, which could make the time ideal to productively address alignment.

## The Algebra Assumption

In the meantime, claims about the value of two years of algebra appear to go beyond what the evidence can bear. Most of the research about the connection between Algebra 2 and college success is based on correlation. As long as Algebra 2 is a college requirement, it is by definition a powerful predictor of students' success. It is possible that statistics or other quantitative reasoning courses could offer equivalent or even stronger preparation for students. The research that does correlate academic success with algebra is typically based on high school course taking, which plainly fails to validate the current system of testing students regardless of their prior course-taking.

If, over time, studies of alternative math sequences show that the students do well in subsequent studies and careers, there will be an empirical basis for the alternatives.

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## EXECUTIVE SUMMARY (Cont.)

## New Approaches to Math Articulation

Math requirements and their articulation across educational segments are in flux in several states: Texas' New Mathways Project is developing remedial sequences leading to Statistics and Quantitative Reasoning. Students may be designated college ready "for any freshman-level math course" or "for non-algebraically intensive mathematics courses." Task forces in Ohio and Georgia concluded that College Algebra should not be the default math requirement outside of fields requiring Calculus, calling for new remedial prerequisites besides Intermediate Algebra. Similarly, a Massachusetts task force on transforming developmental math recommended that community colleges realign their prerequisites with their college-level courses, implying that four-year institutions should accept them. And community colleges in Colorado are implementing a new quantitative literacy sequence in addition to the standard algebraic literacy sequence.

## Looking Ahead

The idea of providing algebra exposure to more students is a compelling one. But, as non-math faculty and administrators increasingly enter the conversation, the question has become whether that idea is compelling enough to dictate-or eliminate-opportunities for some students. Elevated failure rates in courses like Intermediate Algebra and College Algebra make it clear that the futures of hundreds of thousands of students could be at stake.

This could be a false choice for California unless the capacity of the postsecondary system grows to meet the demands of greater college attainment. Absent a change in the state's priorities, the state's universities will not have space for all students who are academically eligible to transfer. Under the status quo, then, math requirements are effectively rationing access to higher education.

Such pragmatic considerations could make it hard to have a serious conversation about a coherent vision for math preparation. But the pragmatic needs of students could make such a conversation impossible to avoid.

The time is ripe for critical conversations about what it means for an educated person to know math.

Ideally, this conversation will look beyond mere curricular alignment to focus on how to ensure that math education in the 21 st century can better serve the needs of students and the state. California's education systems should consider the following:

- Intersegmental conversations are needed to deepen alignment across segments in math education. Rather than abstract discussions, these should be concrete projects, focused on meaningful alignment.
- Given the impressive results of innovations with alternative remedial sequences to date, the higher education segments should not adopt policies that would interfere with the ability to gather evidence on the effectiveness of these alternatives.
- System leaders and state policy makers need to develop greater clarity about the enrollment capacity constraints of California's three higher education systems, so that choices about entrance and graduation requirements can be separated from choices about enrollment capacity.


# DEGREES OF FREEDOM: Varying Routes to Math Readiness and the Challenge of Intersegmental Alignment 

## Foreword

Charles' path from community college to university was a long one. The prototypical transfer student spends two years at a community college. Charles spent 12, earning not one but four two-year degrees. The explanation? Algebra. Charles' favorite subject in elementary school, math, had become his nemesis by high school. Though he earned his diploma, he never felt he understood algebra.

When Charles entered a Bay Area community college in 2001, a math test score placed him into a remedial sequence. After passing the first of four remedial classes, Arithmetic, he once again found himself facing PreAlgebra. Over the next seven years, he attempted it on and off, withdrawing each time to avoid failing. Finally, he tried Liberal Arts Math, a class that, at the time, counted toward an associate degree. Passing that gave him enough credits for three degrees. An honors student, he spoke at his graduation in 2009. But still he hadn't met the math requirements to transfer to a university.

That took another five years. He managed to pass Pre-Algebra and Algebra 1, but Algebra 2 was a
struggle until 2012. That's when he heard about a new remedial class in pre-stats that prepared him for transfer-level Statistics. With those two classes under his belt, he earned his fourth associate degree in 2014. Now at Cal State-Northridge, Charles is working toward a degree in child development. He is pleased that he has had two opportunities to use statistics-and no occasion to use algebra.

Charles' odyssey illustrates how the algebra-intensive math curriculum commonly dictates students' options for entering and completing college, including their ability to transfer from two-year to four-year institutions. Instructors experimenting with alternative math curricula point to stories like his to argue that otherwise successful students who are not pursuing technical fields are better served by preparation for Statistics and Quantitative Reasoning. But critics wonder how students can be adequately prepared for a fouryear degree without at least two years of algebra.

In fact, the correlation between two years of algebra and college outcomes has long bolstered the assumption that algebra is necessary for college success, leading some
equity advocates to promote offering more algebra to all students. As Robert Moses, founder of the Algebra Project wrote:

Algebra, once solely in place as the gatekeeper for higher math and the priesthood who gained access to it, now is the gatekeeper for citizenship; and people who don't have it are like the people who couldn't read and write in the industrial age (Moses \& Cobb, 2001).

Almost half of states require two years of algebra for high school graduation, and the Common Core State Standards being implemented in the majority of states continue that emphasis.

While the intent has been to raise achievement, the hidden underbelly of algebra expectations actually has been swelling enrollment in college developmental (also known as remedial) math over the last few decades. This is especially evident at community colleges, the recent focus of concerted efforts by states and foundations seeking to improve college completion. Those efforts have cast a spotlight on developmental courses as a key "loss point" for students, spawning a host of research and development initiatives seeking to im-
prove the success of students needing remedial courses.

After nearly 10 years of investment, the initiatives with perhaps the most promise to move the needle on students' math success are also the most controversial. Alternative developmental math sequences have been designed for students interested in nonalgebra intensive fields. Early results at community colleges have been extraordinary, opening up opportunities for students such as Charles. But their emphasis on statistics and quantitative reasoning defies long-held beliefs about the importance of algebra for all students, particularly on the part of math departments to which universities typically defer in setting general education requirements.

Universities' wariness about the initiatives creates a dilemma for students seeking to transfer to universities, leaving community colleges hesitant to make alternatives available. It also reveals the predicament of community colleges: Poorly funded institutions at the crossroads between high school and higher education, they are expected to serve students who were ill-prepared in high schoolimproving their success rates while accepting the standards (and capacity constraints) of four-year universities. And it underscores the need for more clarity and consistency in math expectations.

In a 2003 article, Articulation and Quantitative Literacy: A View From Inside Mathematics, mathematician Bernard Madison observed:

In no part of U.S. education are the problems caused by disunity (or lack of articulation) greater than in mathematics.... A principal cause of the transition problems in U.S. mathematics education is the lack of an intellectually coherent vision of mathematics among professionals responsible for mathematics education. Mathematicians similarly lack a coherent vision.

The issue is particularly salient in California, given the state's history of experiments with math reforms and its role as the incubator of several alternative math curricula. With the implementation of new math standards intended to improve readiness for college, understanding the intersection of those standards with postsecondary expectations for admission and transfer is critical, especially because developments here often reverberate across the nation.

LearningWorks and PACE are disseminating the Degrees of Freedom series to highlight these issues and stimulate improved policies. The first report in the series highlighted a profusion of experiments with changing the math expectations across the spectrum from high school through various college majors and into medical
schools. This, the second report, will highlight experiments with alternative remedial math sequences at community colleges in California and the particular challenges of aligning them with four-year university requirements for students seeking to transfer from community colleges. It also will examine math alignment from high school through college, revealing an underlying misalignment of existing requirements, and show how the resulting restrictions serve to ration access to higher education. Finally, it will surface some recommendations for improving the status quo.

## The Transfer Imperative

By design, improving college completion in California requires attention to the transfer route to a bachelor's degree. Under California's 1960 Master Plan for Higher Education, freshman slots at universities were limited to the top one third of high school graduates, leaving community colleges to broaden access to higher education. With their doors open to all high school graduates, the colleges were tasked with providing lowerdivision education to students who, due to eligibility limits, could not be admitted directly to the universities from high school. Sixty percent of university students were to be juniors or seniors, and therefore one of every three admits was to be a transfer student.

As demand for higher education increased and community college enrollment grew, transfer gained a vital position in California's higher education landscape. In fact, a higher proportion of California undergraduates starts at community colleges than in other states. Around 29 percent of University of California graduates and 51 percent of California State University graduates began their postsecondary education at a community college, with upwards of 60,000 students transferring each year to the two public university systems and nearly 30,000 more to private institutions.

From the universities' perspective, the transfer process appears to work well. Nationally, students who transfer are just as likely to earn bachelor's degrees as are juniors who started at a four-year university, in spite of losing some credits due to articulation problems (Melguizo et al., 2015; Monaghan \& Attewell, 2015). The trend is similar in California. In recent years, transfer students' graduation rates have been slightly higher than those of native juniors at CSU and slightly lower at UC. As a result, the universities have had little incentive to alter the process.

However, according to a 2014 report, most UC campuses are falling short of the 33 percent target for transfer admits, and the UC system's new president, former U.S. Home-
land Security Secretary and Arizona Governor Janet Napolitano, has pledged that improving the transfer process will be a top priority. All but one of the campuses not meeting the target have committed to increasing that share, but UC leaders say that will require additional funding because upper-division enrollments are more costly than lower-division. CSU officials say their system is hewing close to the targets so far.

Community colleges have faced mounting pressure to increase student success rates, amid scrutiny over the relatively small proportion of students who go on to transfer. Though transfer rates are difficult to define with precision, because not all community college students intend to transfer, rates generally appear low; some estimates find that fewer than a quarter of degreeseeking community college students in California ultimately transfer to a four-year university. Moreover, while UC's leaders once envisioned that transfer students would bring racial diversity to the university, incoming transfer students have become less diverse than freshman classes. They are also less geographically diverse: Half come from just 19 of the state's 112 community colleges.

Changing that is made more challenging by the "zero sum dynamic" inherent in the Master Plan (Geiser \& Atkinson, 2012). With demand
for freshman seats high, the number of students transferring to California's two public university systems has changed little over the past 12 years, even as the number of "transfer ready" students has climbed steadily, suggesting that community colleges are preparing more students than the universities can accommodate. New "associate degrees for transfer" may further strain enrollment capacity at the four-years.

A recently approved experiment to allow a few community colleges to offer four-year degrees may be a partial solution to ensuring access to four-year degrees. But at least under current policies, the vast majority of students will still need to pass remedial, or developmental, math. A variety of approaches is being attempted to ensure that they do. But in an illustration of math's role as a sorting mechanism, the more these new approaches succeed, the more the capacity limits may come into play.

## Developmental Math Reforms

Most of the community college reforms directed at improving student success, including transfer, address developmental education in some way, because remedial courses represent a major barrier to students' progress. That is especially true in math. Completion of a general education math course is one of the

## RESEARCH YIELDS NEW INSIGHTS ON DEVELOPMENTAL MATH

As described in LearningWorks' 2013 report, Changing Equations: How Community Colleges Are Re-Thinking College Readiness in Math, a concerted focus on how best to prepare students in math has produced new understanding about the role of placement exams and remedial sequences. Below is an updated recap of the relevant research.

## Hurdle for the Majority

- Nationally, 68 percent of two-year college students place into remedial math.
- In California, an estimated 85 percent place into a math course below transfer-level (but students pursuing an associate degree don't require transfer-level math)
- Though enrollment in remedial education is consistent across all racial groups, under-represented minority students were more likely to be placed at lower levels: $61 \%$ of African Americans and $52 \%$ of Latino students were placed into Arithmetic or pre-Algebra, while only a third of white and Asian students were placed that low.

Sources: Hodara, 2013; California Community Colleges Chancellors' Office, 2012; Perry et al., 2010

## Most Never Graduate

- Nationally, only 32 percent of students assigned to developmental math (some of whom actually skip developmental math) ever complete a college-level math course that is typically required for graduation.
- In California, 30 percent of students who enroll in developmental math complete a math class required for transferring to UC or CSU within six years.
- The lower in the remedial sequence students begin, the less likely they are to complete a transferable class necessary to graduate.

Sources: Bailey, 2010; California Community Colleges Chancellor's Office, 2013

## No Positive Effects

- Regression discontinuity studies at two-year and four-year systems around the country (Florida, New York, Virginia, and Texas) "have largely found that developmental math has an insignificant or negative effect on the educational outcomes of students."
- In addition, developmental math credits were found to have a negative effect on labor market outcomes.

Sources: Hodara, 2013; Hodara and Xu, 2014

## Mediocre Instruction

- Qualitative research found that remedial math instruction often does not incorporate the type of instruction found to improve student learning. Rather, it frequently entails "extremely tedious" coverage of skills and procedures "presented without any justification for why such skills might be useful" as well as "an emphasis on getting the right answer, rather than any conceptual understanding of why an answer is correct."

Source: Grubb, 2012

## Placement Exams are Weak Predictors

- About one quarter of students placed in remedial math could have passed a college-level course. Using high school grades instead of test scores reduces "severe error" rates by 30 percent. After using high school grades, placement exams provide little to no incremental improvement (i.e., they explain a maximum of 6 percent of the variation in course outcomes).

Source: Scott-Clayton et al., 2012

## Not Relevant?

- The typical remedial math sequence culminates in Intermediate Algebra, a course that prepares students for Pre-Calculus and Calculus, which are primarily required for Science, Technology, Engineering, and Mathematics (STEM) majors. Only 20 percent of community college students and 28 percent of four-year university students choose STEM majors. One estimate indicates that between 18 and 31 percent of bachelor's degree holders use Algebra 2 or beyond on the job. Another found that 19 percent of employees use any algebra in their work. Moreover, increasing numbers of students require the use of statistics and quantitative reasoning, topics not covered by typical remedial sequences, in college and career.

Sources: NCES, 2014; Georgetown University Center on Education and the Workforce-personal communication 5/28/13; National Research Council, 2008
most significant milestones to transfer (RP Group, 2010). But few students start community college ready to take such a course.

The vast majority of community college students are placed into remedial sequences typically consisting of one to four courses. Those who enter unprepared for a college-
level math class rarely get a chance to take one, and even fewer manage to transfer, a reality that has sparked research and development to improve students' success with developmental math in California and nationally.

As discussed in a prior LearningWorks report, Changing Equations
(Burdman, 2013), the emerging research is calling into question the assumptions on which the current system is based (see Box, Research Yields New Insights). The emerging insights include:

- Math is a hurdle for the vast majority of community college students;
- Most students deemed "unready" in math will never graduate;
- Mediocre math teaching is common in community college developmental math courses, with studies suggesting that the courses have insignificant or negative effects on students' educational outcomes;
- The tests community colleges use to determine readiness "underplace" a significant proportion of students; and
- The math sequence required by most colleges is irrelevant for many students' career aspirations.

These revelations depict inaccurate community college tests unjustly placing large numbers of students into dull and difficult courses that determine their futures regardless of their relevance to students' aspirations. Increasingly aware of the dilemma, community colleges around the country have begun to pursue a variety of math readiness reforms.

Instructional reforms, including contextualized instruction and modularized courses, aim to replace the "drill and kill" mode of teaching with more effective pedagogy (Grubb, 2012). A range of placement reforms seeks to change the role of placement exams in determining students' readiness for college-level courses. These include efforts to im-
prove the tests as well as policies to acknowledge their limitations by deemphasizing them (Burdman, 2012). Finally, experiments with curricular pathway reforms aim to make students' math sequences more relevant to their course of study, more reflective of the ways math is used in contemporary disciplines, and more conducive to deeper learning (Burdman, 2015).

These alternative remedial pathways or sequences are directed at students who don't major in STEM fields, currently about 80 percent of community college grads. The typical remedial math pathway includes Elementary Algebra (which, like Algebra 1, introduces the basic idea of using abstract notation to represent quantities and solve problems) and culminates with Intermediate Algebra (which builds on these concepts with more complex formulas and manipulations). Most non-STEM students will not use algebra at that level, because most don't need to take Calculus. While basic algebraic concepts are useful for many students, most students do not need some of the more advanced topics in Intermediate Algebra unless they plan to take Calculus.

The alternative sequences typically begin with a remedial course that replaces some of the algebra material with content that prepares students for Statistics or Quantitative Rea-
soning. They culminate with a col-lege-level course in either Statistics or Quantitative Reasoning, instead of the typical College Algebra or Pre-Calculus course (see Box, Standard vs. Redesigned Math Pathways, p. 11).

Each type of reform is designed to promote student progress through the pathway, and each has shown some promise in early studies. For example, efforts to accelerate students through the standard remedial math curriculum have led to improved completion rates (Jaggars et al., 2014). And the proportion of students who completed collegelevel math increased after Virginia introduced a new placement test (Rodriguez, 2014). These and other placement reforms will be discussed further in the third report in this series.

In combination, the approaches appear to be particularly promising. The new curricular pathways tend to integrate instructional improvements as well as placement reforms. Based on early studies by the Carnegie Foundation for the Advancement of Teaching and others, students in the new sequences are three to four times as successful in passing collegelevel math requirements as students in standard remedial sequences over a similar or shorter time period (see Box, Early Success, p. 12).

# Standard vs. Redesigned Math Pathways 

|  | REMEDIAL COURSE | TRANSFERABLE COURSE | EXAMPLE |
| :---: | :---: | :---: | :---: |
| STANDARD PATHWAY | Sequence Cumulating in Intermediate Algebra | College Algebra, Pre-Calculus (Statistics, Liberal Arts Math, \& other alternatives) |  |
| REDESIGNED REMEDIAL PATHWAY | Prep for Statistics or Quantitative Reasoning | Existing Statistics or Quantitative Reasoning Course | Pre-statistics courses offered through the California Acceleration Project, Acceleration in Context, and a few CSU campuses |
| REDESIGNED PATHWAY | Prep for Statistics or Quantitative Reasoning | New Statistics <br> or Quantitative Reasoning Course | Carnegie Foundation's Statway and Quantway Programs and Humboldt State University's Math 103i/43 sequence |

Such effect sizes are virtually unheard of in the literature on improving community college student outcomes. "We were somewhat surprised by it," noted Terrence Willett of the Research and Planning Group, author of one of the studies. Even allowing that there are limitations to the studies to date, the dramatic results suggest that the experiments deserve to be continued and the participants followed for several more years to understand how they fare after transferring to four-year
universities. But whether they actually can transfer has not been clear.

## California Conundrum: <br> University Limits on Transferability of Reforms

For the community college reforms to be robust and scalable, they need to be acceptable under universities' articulation policies. Alternative pathways, however, have run into difficulty, because they tread on universities' territory in setting lowerdivision requirements. The resulting
uncertainty about whether students in them can transfer smoothly has frustrated efforts to expand them in some places.

Without assurance that universities will accept the pathways, colleges have understandably been wary of offering alternatives to transferbound students. One community college official likened the situation to Allied troops being dropped in the surf at Normandy only to be gunned down by enemy forces during World War II. He said that

## EARLY SUCCESS: NEW REMEDIAL MATH PATHWAYS

Developed by the Carnegie Foundation for the Advancement of Teaching with a goal of improving success in remedial math sequences for students who are not pursuing majors requiring Calculus, Statway was launched more than two years ago as a two-course sequence encompassing both remedial and college-level content. In year two, it involved 19 community colleges and four state universities and enrolled about 1500 students across five states. Only one campus has scaled the program beyond a few sections.

More than half of students who started Statway in Fall 2012 successfully completed it in the spring. This compares very favorably to the 6 percent of non-Statway remedial math students who manage to receive credit for col-lege-level math in their first year. In fact, only 15 percent achieve this goal after two years, so Carnegie says that the program tripled success rates in half the time. Similarly, Quantway's first course,
a remedial quantitative reasoning course, doubled the success rate of remedial course completion in half the time (Van Campen et al., 2013). Carnegie has not yet shown that the program can be scaled to serve large numbers of students at a single college.

Other experiments to redesign remedial sequences have also shown remarkably strong results. For example, an independent evaluation of the California Acceleration Project's alternative math pathways at eight colleges found "large and robust increases" in gatekeeper course completion. In math, students in the program were 4.5 times more likely to complete a transferable course than students in traditional remedial sequences (Hayward \& Willett, 2014). Unlike Statway, which costs $\$ 50,000$ per campus to offer, there is no charge for colleges to participate in CAP.

A more recent initiative, called the New Mathways Project, led by the Charles A. Dana Center at the Uni-
versity of Texas-Austin, is working with community colleges in Texas to diversify remedial sequences. As of Fall 2014, 20 colleges had begun implementing the model. Preliminary results mirror those of the earlier studies, according to an April 2015 report by MDRC.

An experiment conducted by the City University of New York found that most students who placed into elementary algebra were able to succeed in a collegelevel Statistics course with extra support in the form of a workshop. In fact, they outperformed students in a remedial elementary algebra class, with or without the workshop. Due to ethical considerations, the randomized control trial did not examine how the students would perform in the Statistics class without extra support. However, the findings are consistent with the other experiments to date.
in the interest of students because of transfer policies," said Bruce Vandal, vice president of Complete College America, an advocacy group that favors alternatives.

Community colleges in California are among those that have faced this conundrum. The state is home to some of the earliest experiments to improve college completion by redesigning remedial math pathways, so much of the stunning success started here. But, as if they are victims of their own success, the experiments have faced some of the toughest scrutiny from universities here in California (Wong, 2013). That scrutiny has threatened the future of the experiments. It's not just a matter of credit roadblocks; it has been a question of whether students can transfer at all. California has the added challenge that limits on four-year enrollment capacity could thwart attempts to relax the requirements.

At the heart of the dilemma is how the universities interpret their common rule that the undergraduate math courses required for transfer must have a prerequisite of Intermediate Algebra. Students attending community colleges, therefore, are expected to show proficiency in Intermediate Algebra by passing either a placement exam or an Intermediate Algebra remedial course before enrolling in a college-level math class. The assumption is that Intermediate Algebra, which typically covers topics such as polynomials and quadratic equations, corresponds to high school Algebra 2, the highest math course required for
admission to UC (and CSU) as a freshman.

Behind that stipulation is the feeling of many math professors that higher level algebra is essential to becoming an educated person. "The assumption that the only purpose of developmental math is to get someone ready for general education quantitative reasoning courses is wrong," said David Bao, chair of the math department at San Francisco State University. "Developmental math, if done properly, allows its veterans to reap lifelong benefits in numerous instances."

Reflecting their unique missions, each university system treats students' math skills differently upon entry, leading to different approaches at the transfer level.

## UC system transfer requirements.

 As one of the most selective public universities anywhere, UC, by definition, maintains stringent admissions criteria for freshmen and transfer students. Relaxing those criteria could cause the number of eligible students to exceed available spaces."This is one of the problems with being UC," noted George Johnson, a UC-Berkeley professor of mechanical engineering and former chair of the statewide committee that sets admission requirements. "Our charge in the Master Plan is
to draw from the top one eighth of high school graduates and to admit transfer students at the rate of one third of our graduates. That necessarily excludes some students."

UC and CSU require entering freshmen to have completed a common pattern of high school courses. Since 1984, the UC system has stipulated that entering transfer students must meet the same math and English requirements as entering freshmen. In math, that has been Algebra 1, Geometry, and Algebra 2. The policy is designed to ensure fairness and consistency, preventing a student from circumventing freshman eligibility requirements by spending two years at a community college, for example. As K-12 schools implement new Common Core math standards, UC has revised the requirement to equate with the new high school math content, which encompasses most of the traditional Algebra 1-Geometry-Algebra 2 sequence plus other topics such as probability and statistics.

In accordance with this change, UC's Board of Admissions and Relations with Schools (BOARS), comprised of faculty, has reviewed two types of alternative remedial sequences. On the first review, both types-a Statway sequence consisting of two newly designed courses and a California Acceleration Project (CAP) course that pairs an existing sta-
tistics course with a new remedial course-were found lacking.

On the basis of a more recent review requested by five community colleges implementing Statway, BOARS recently gave approval for Statway as implemented at those colleges. But their reasoning was not fully clear, leaving questions about the implications for other statistics pathways offered by far more colleges. The decision also does not apply to CSU, which admits three to four times as many transfer students annually as UC.

CSU system transfer requirements. CSU's policies on transferable math look like UC's, at least on paper. CSU also has begun to have a transfer squeeze similar to UC's, which may explain why the system has tightened its math requirements in recent years. According to a 2011 executive order from CSU's chancellor, general education math courses must have an "explicit Intermediate Algebra prerequisite." That drives the system's policy on transfer students, and explains why a course like Introductory Statistics, which does not require much algebra, still must list Intermediate Algebra as a prerequisite to be transferable.

However, Cal State's interest in remediation is quite different from UC's. Despite admitting the top one third of high school graduates, CSU
historically has faced high remediation rates among its own freshmen. Though the roughly one third of CSU students who require remediation is far less than the estimated 85 percent at community colleges, it's enough to give CSU's math departments a nuanced experience with developmental math.

Nearly 20 years ago, the CSU system became one of the first in the nation to embark on an effort to reduce remediation rates. Though still far from a 1996 goal of reducing the proportion of students needing remedial courses to 10 percent, CSU has adopted a range of policies that have produced notable strides in reducing the need for remedial math and modest progress in improving success for those students who still require it (see Box, Twenty Years and Counting, p. 15).

The CSU system also has shown some interest in adopting alternative pathways. In 2010, it granted provisional clearance for its campuses to accept one pathway that does not include Intermediate Algebra: As part of a study, it is allowing campuses to enroll transfer students who participated in the Carnegie Foundation's Statway program. In addition, two CSU campuses (San Jose and Sacramento) are currently running Statway experiments for their own remedial math students.

But Statway's experience at CSU has been rocky, due to the ambivalence of many math faculty. Of the five campuses that had originally signed up to offer the program, one decided against offering it and two others abandoned the experiment due to the dissatisfaction of math faculty. A few others are considering offering the program outside of their math departments (as San Jose has been doing).

In late April, the math chairs of CSU's 23 campuses collectively released a resolution harshly critiquing Statway. "We do not support the continuation of Statway as a venue for side-stepping remediation at community colleges, and do not recommend the replacement of Elementary/Introductory Statistics courses at CSU campuses by Statway," the chairs wrote. They listed numerous arguments for this position, including:

Statway "limits its coverage of developmental math to arithmetic and straight lines (and optionally a bit of exponential functions) in the context of regression analysis. Therefore, the underlying mathematical content is not thoroughly presented for assimilation by the students."
"In granting someone a university degree it is a matter of honesty to make sure that they meet the de-

## TWENTY YEARS AND COUNTING-THE CSU REMEDIAL MATH EXPERIENCE:

It has been 20 years since the CSU system began facing down its remedial education challenge. In 1995, the system's trustees took aim at remedial courses with a plan to eliminate them. A year later, they passed a softened version of that stance, which sought to eliminate the majority of remedial classes by 2007. Though nothing approaching that has come to pass, a series of measures appears to have made a major dent in the proportion of students requiring remedial math,
reducing it from 54 percent in 1998 to 29 percent in 2014.

These measures include making placement tests mandatory for all nonexempt freshmen, giving stu- dents only one year to take their remedial courses, using an 11th- grade math test to signal to stu- dents whether they are on track for required collegelevel math courses, and mandating that students requiring remediation begin their courses during the summer before they enroll.

Roughly half of incoming students are deemed college-ready in math based on tests taken during high school, such as the SAT or Early Assessment Program test. Still others pass the placement test, leaving around a third of students who are not proficient. Roughly 4 percent satisfy their remedial requirements during the summer before school starts, leaving 29 percent to start in remedial math courses.


Source: Author's analysis based on published reports
gree requirements. CSU should label someone remediation-free only if they truly have the credentials as set out by ELM [CSU's placement test]."
"Using pathways such as Statway to sidestep remediation, or claiming that they can do as good a job as a full dose of developmental math, is unethical and irresponsible."

Compared to community colleges, CSU campuses have a more manageable remediation challenge. CSU students are better-prepared than community college students and the attrition rates less stark: Around 89 percent of students who test below proficient in math and/or English survive remedial sequences, though they are given only one year in which to do so. Remedial math, therefore, is less of a deterrent to graduation.

And at least some campuses are making progress in tackling it without resorting to alternatives. CSUMonterey Bay, for example, has seen huge improvements in remedial math success: Its new remedial sequence helped lift campus-wide first-year retention rates from 65 percent to 80 percent, with remedial students even higher at 90 percent.

Another obstacle for CSU in accepting alternative remedial prerequisites is the dilemma of how to treat students who have taken alternative
remedial sequences and subsequently decide to pursue STEM majors. Under system policy, these students are considered college ready, so it is not a simple matter to offer them an additional remedial or bridge course to prepare them for STEM courses.

Lastly, as discussed below, some CSU campuses have also been experimenting with their own alternative courses. Math faculty believe they do a better job covering traditional remedial material.

## Crucibles of Confusion: Colleges Face Multitude of Math Expectations

University limits on transferability are indicative of the predicament faced by community colleges: As key transit points for students moving through the higher education system, from high schools to universities, the colleges are crucibles for an array of expectations that are often confusing, ambiguous, or altogether conflicting. The effort to satisfy those requirements may ultimately hinder their effectiveness in serving students.

Developmental math reforms are illustrative. Two-year colleges have been under intense pressure over the past decade to improve transfer rates and other outcomes. This is a difficult task given the weak preparation of many of their students as well as colleges' poor funding, not to men-
tion limited space at four-year universities. Foundations and advocacy organizations interested in improving student outcomes have pressed for new models. And yet one of the most promising of those models is at odds with the requirements of both the high schools whose graduates they serve and the four-year universities they prepare students to attend.

The challenge for community college math departments goes beyond the narrow question of how to treat alternative sequences to the broader one of how to make changes that improve student success: Should their focus be on re-teaching high school material, preparing students for their subsequent majors, improving remediation rates, aligning with Common Core, meeting university freshman entrance requirements, or deepening students' quantitative literacy? In an ideal world, these goals would not conflict with each other. But, in reality, they can.

Faculty members teaching the new alternatives say they are mainly looking to prepare students for subsequent study, and they have been energized by the results. In addition to passing their college-level gatekeeper math courses, the instructors believe that students are learning mathematics at a deeper level than in traditional courses. Says Myra Snell, whose course at Los Medanos

College in Pittsburg, CA is being replicated in California and nationally:

The thinking students are doing in pre-statistics work far exceeds anything that we're currently doing in traditional algebraic instruction. Students are presented with open-ended problems where they have to analyze real data to make decisions in the face of uncertainty. I think the foundational skills are not in memorizing algebraic procedures, but in thinking quantitatively, being able to deal with data, understanding things like rates of change.

At least some students agree. Like Charles, Jodie found algebra a challenge during high school. She returned to college in her thirties and, like Charles, was a star studentexcept in math. Faced with a series of four remedial courses, she passed the first two (Arithmetic and Pre-Algebra) on the first try, but Elementary Algebra stumped her, and the teacher's explanations didn't seem to help. An alternative math sequence for students interested in non-technical fields opened up a route to transfer. Within two semesters, she had completed the two-course sequence, even surprising herself with a perfect score on her stats final.

Today, a student in speech and language pathology at San Francisco

State University, she is grateful for the alternative sequence, and especially the remedial course that substituted for Intermediate Algebra. "Statistics made more sense to me than algebra," she recalled. "It was really valuable. The understanding I got from the pre-stats class has been exceptionally relevant for the journal articles I'm reading now in my major. Everyone should take that class regardless of whether they're going to take statistics."

Faced with such testimonials, enterprising math faculty members, who feel they are doing students a great service, say it makes little sense that the universities have questioned the appropriateness of the sequences. At a 2014 conference on student success, a group of them lamented that irony.
"Our entire sequence is designed as replicating the high schools," noted Michelle Brock, a math instructor at American River College who is participating in Statway." "It's not actually designed for success in our classes. We need to take this opportunity to look at where our college-level courses are and look at our students and provide them tools to give them a fighting chance to be successful."

Another challenge is that the way remedial courses are coded in colleges' computer systems doesn't make allowances for alternative
content or for courses that accelerate students through more than one level of math. This makes the success of alternative pathways harder to document without costly external evaluations. "As far as getting students through remediation, this has been an incredible intervention, but we can't track it statewide," said Janet Fulks, a biology instructor and student success dean at Bakersfield College. "We need to find a way to code these courses. It's important stuff."

To address the transfer dilemma, most colleges experimenting with alternatives have avoided listing prerequisites besides Intermediate Algebra. Instead, some 30 California community colleges are offering statistics pathways via a loophole: Existing regulations allow students to challenge a prerequisite course by showing that they can pass the destination course without it. However, the reliance on this workaround along with the uncertainty about long-term transferability makes the programs hard to scale. And the workaround can't help programs like Statway, in which the college-level course has also been re-designed, until UC's recent policy change.

## Math Readiness and Its Elusive Definition

If anything, the discussion of alternatives has laid bare considerable
ambiguity about existing math requirements. Despite an increasing consensus that students need more grounding in statistics and data analysis and a myriad of experiments taking place across the country, many higher education institutions maintain the traditional emphasis on two years of algebra. Unfortunately, the Algebra 2 that most high schools, colleges, and universities collectively view as the minimal culmination of students' math preparation is ill-defined at best (Loveless, 2013). This raises the question of whether the alternative courses' visibility means that they are unfairly bearing the brunt of the scrutiny. And it points to a need for new approaches to improve curricular alignment and transparency.

Ambiguity over alignment. Initiatives to improve college-readiness have emphasized the need for higher education institutions to be clear and consistent in describing their expectations in order for $\mathrm{K}-12$ schools to have a chance of meeting them. As Mike Kirst, a retired Stanford University education professor and current president of California's State Board of Education wrote, "The more that incoherent and vague signals are sent by universities to students, the less adequate student preparation for higher education will become" (Kirst, 2003).

Unfortunately, statewide alignment in math expectations appears to exist more in name than in practice. In interviews, the majority of college and university math faculty were unclear about the content their students were learning in other segments, even before adoption of the new K-12 standards. For example, more than one UC faculty member who believes it is important for all students to take Algebra 2 acknowledged that he isn't intimately familiar with the course's content.

In fact, the field has lacked precise nomenclature for describing alignment and especially for distinguishing the list of topics covered by a course from the depth and sophistication of students' learning. Content is only one dimension of rigor. For example, the SAT is considered to include very little Algebra 2 content-only 3 percent of the questions, according to one analysis (Kirst, 2003). At the same time, most experts agree that students who have not taken Algebra 2 are unlikely to perform well on the test because they haven't mastered Algebra 1 at a sufficiently sophisticated level.

While Algebra 2 and Intermediate Algebra are supposed to be equivalent, faculty provide conflicting accounts of the consistencies between them. Some view college Intermediate Algebra as a more challeng-
ing course, based on the fact that most students who pass Algebra 2 in high school are placed into remedial courses at community college. Each year, tens of thousands of California students who have passed Algebra 2 in high school test into one or more remedial courses at community colleges.

Others say that, when it comes to content, high school Algebra 2 actually covers more. "Community colleges have something of a tailored approach to Intermediate Algebra," acknowledged Beth Smith, a community college math instructor and former chair of the community colleges' statewide faculty senate. "The high schools have the advantage of more time. They have the students for 36 weeks," whereas college courses are just one semester.
"There's a big miscommunication about what people mean by Intermediate Algebra," notes Judy Kysh of San Francisco State University, the rare university mathematician who has studied math curricula across segments. "The courses I've seen in most places basically treat remedial Intermediate Algebra as advanced Elementary Algebra. They don't deal with a lot of the material that is in a normal high school Algebra 2 class. Looking at textbooks, College Algebra comes closer to what is actually in high school Algebra 2."

## DEFINING INTERMEDIATE ALGEBRA VS. TRADITIONAL ALGEBRA 2

## Entrance <br> Remediation

UC Intermediate Algebra = high school None
Algebra 2 content (no test)

| CSU | ELM tests about 40 percent of tradi- <br> tional Algebra 2 content | Remedial courses cover zero to 100 percent <br> of high school Algebra 2 content |
| :--- | :--- | :--- |
| Community colleges | Each college chooses its own tests and <br> sets its own cut-off scores | Remedial courses cover varying subset of <br> Algebra 2 |

Uneven scrutiny of alternatives. The lack of clarity about the definition of second-year algebra is problematic given its role as a standard for college readiness at public universities in California and elsewhere. However, the ambiguity has gone largely unnoticed when it comes to community college transfer, because UC and CSU typically review the college-level courses intended to transfer, not the remedial courses that may serve as prerequisites.

The scrutiny of alternative remedial sequences in California came about because of two sets of unusual circumstances:

## - The Carnegie Foundation's

 Statway program consists of two courses, a remedial pre-stats course and a college-level Statistics course. Because the collegelevel course was new, it needed to be approved for transfer tothe universities. And because the course is part of a two-course sequence, the universities looked at the entire sequence. On its initial review, UC found the sequence unacceptable, but recently reversed course and decided to accept it. CSU agreed to accept the transfer sequence, only temporarily, as part of an experiment that is set to conclude in 2016.

- In submitting a college-level statistics course for routine articulation to UC in 2009, Los Medanos College listed its pre-stats class along with Intermediate Algebra as prerequisites. The course was approved that way, but in 2012, UC scrutinized it due to complaints from a community college math instructor and sys-tem-wide faculty leader. After reviewing the pre-stats remedial class, UC's BOARS determined that the statistics course was no
longer acceptable, even though the content of that course had not changed. The college subsequently removed the alternative prerequisite course, because the loss of articulation would effectively have removed Statistics as an option for students, regardless of whether they had taken the alternative class or not.

UC and CSU officals have stated that they don't intend to make a habit of reviewing remedial math courses. However, given the wide range of courses carrying the name Intermediate Algebra, it is clear that others could be found lacking if reviewed. By default, prerequisites with Intermediate Algebra in their titles have generally been assumed acceptable, and some alternatives have kept Intermediate Algebra in their titles just to retain transferability. Other bet-ter-known alternatives have drawn scrutiny because they telegraphed
their plan to reduce Algebra 2 content in order to teach pre-statistics. In its recent approval of Statway, UC signaled a move toward focusing on course content instead of titles.

In fact, various alternative approaches are being implemented at universities, including the CSU system, with little fanfare or controversy. In 2002, CSU revised its Entry Level Mathematics (ELM) test. Based on a two-year review by mathematicians and math educators, the new test focused on the skills needed in general education quantitative courses, rather than just those needed by students planning to pursue quantitative majors. That meant less emphasis on algebraic manipulation. According to a 2009 analysis by Achieve, a Washington, DC-based education reform organization, the ELM covers roughly 40 percent of traditional Algebra 2 content (Achieve, 2009).

That change led to an immediate 18 percent drop in the proportion of freshmen sent to remedial math courses. It also opened the door for CSU's 23 campuses to rethink the content of their highest level remedial math course. Not all such courses at CSU campuses appear to meet the "explicit Intermediate Algebra" standard to which community college transfer students are held.

At Cal State-Northridge, for example, the remedial course content mirrors the content of the ELM exam, so students aren't expected to master all of the content of traditional Algebra 2. Cal State-Bakersfield redesigned its remedial sequence about 10 years ago to focus on preparation for Statistics, since the majority of students requiring remedial courses went on to Statistics, not College Algebra. Its two-course remedial sequence spans arithmetic through pre-statistics, including some algebra, but little Algebra 2.

Humboldt State's remedial options have an even stronger resonance with the community college alternatives. The campus started offering a two-course quantitative reasoning sequence about 15 years ago, around the same time the ELM revisions were being developed. The sequence is open to students who do fairly well on the ELM, but below the cut-off for college-level courses. These students would be reasonably proficient in number sense, data analysis, and elementary algebra, but not necessarily in Algebra 2.

Several other CSU math departments are also redesigning pathways. These include two that initially considered implementing Statway, but felt that its algebra content didn't rise to the level of the ELM, already a compromise for many math professors. Northridge has developed
an alternate pre-stats course for students who have passed out of Arithmetic. The campus may also create an accelerated Statistics course. San Francisco State University is revising its sequence to emphasize more data analysis and group problem-solving while de-emphasizing symbolic manipulations such as factoring quadratic equations. Noted San Francisco State University math professor Eric Hsu:

The thinking is that a lot of the students end up needing statistics more than they need raw algebraic power. We've been tracking our students in the classes that come later, and basically they are doing as well as or better than the students who didn't get this treatment. Students forced to study fairly technical material that they don't care about are not going to remember it, so they're not going to be significantly better off than students who didn't study it. The vast majority of students placed into remedial math are not students who are going to be hurt by taking an alternative approach.

Seeking alignment and transparency. The inconsistency and ambiguity surrounding the requirements suggest the need for more alignment and transparency. Historically, however, attempts to promote stonger alignment have fallen flat, partly because there are few levers for doing
so. The Intersegmental Committee of the Academic Senates (ICAS) periodically brings together faculty from all three segments to develop shared readiness expectations. Those published recommendations, however, do not seem to have a major impact on how colleges and universities actually treat math readiness. In fact, most higher education math professors don't know they exist.

Some mathematicians who helped craft the recommendations as well as others who have assessed them see them as more aspirational than realistic for all students. "College mathematicians are inclined to describe the student they would prefer to teach rather than the student that is possible and practical to find within the education system," noted Bernard Madison, a mathematician at the University of Arkansas who has led projects for the National Research Council and the College Board's Advanced Placement program.

Another attempt to develop shared readiness standards was the 2010 California Diploma Project. ${ }^{1}$ Through that effort, all three systems agreed that an augmentation to the 11th-grade standards test used by CSU to assess readiness for col-

[^0]lege could serve as a shared indicator of preparedness for non-remedial credit-bearing work. Since that time, 69 community college math departments have begun using the assessment to waive placement testing for students who reach a certain score. However, the agreement was not widely disseminated, and therefore ultimately did not foster deeper alignment conversations across the three systems. (The indicator is now expected to change in conjunction with Common Core-aligned tests currently being introduced.)

The lesson from this limited success may be that, rather than an enforcement action such as that taken by BOARS, or quasi-official proclamations such as those of ICAS and the Diploma Project, there is a need for deeper and more prolonged dialogue to generate pragmatic understandings about alignment. California was the first state in which every higher education system endorsed the Common Core standards, but their introduction has yet to become a vehicle for seeking alignment or fostering transparency among the three segments.

Finding common ground in Common Core. Going forward, many involved with setting readiness standards are hopeful that the Common Core math standards can provide a more sensible benchmark for math readiness than any particular course
name. The standards seek to address the dichotomy between content coverage and level of rigor by including a set of mathematical practice standards. The practices include expertise in areas such as reasoning abstractly, constructing arguments, and attending to precision. However, for now, the standards don't include clear ways of measuring the degree to which a given course develops the practices. Faculty looking to determine the equivalency of their courses have found it easier to focus on readily available content lists.

Even the content lists are the subject of some confusion. In fact, UC's BOARS unwittingly set off a conversation about alignment in late 2013, when it attempted to clarify its policy that, going forward, the standard for math readiness would be the Common Core high school math standards, not an Intermediate Algebra prerequisite. The ensuing confusion took BOARS by surprise. Community college instructors weren't sure whether to replace Intermediate Algebra with the new high school Algebra 2 content or to change their entire remedial sequence, down to Arithmetic and Pre-Algebra, to match Common Core.

Even more confusing, BOARS claimed that Common Core basic math standards contain less Intermediate Algebra content than community colleges typically teach, os-
tensibly freeing community colleges to reduce the Intermediate Algebra content of their remedial courses. But most community college math instructors drew the opposite conclusion, saying that the new standards cover more material than their existing remedial courses.
"When BOARS said they would expect all of the basic standards to be prerequisite to any transferable course, they were essentially making no community college course acceptable for transfer," said Bruce Yoshiwara, a former math professor at Los Angeles Pierce College and former vice president of the American Math Association of Two Year Colleges. "Even our Calculus classes do not include all of the standards."

As a member of a state-level committee that evaluated the Common Core math standards and developed frameworks for California, Yoshiwara conducted his own analysis of the alignment with community college math sequences. This analysis, along with one performed by the firm WestEd for the community college system both found that the algebra content within Common Core exceeds that of the standard Intermediate Algebra class as well as the ICAS expectations (Yoshiwara; Booth et al., 2014).

The fact that community colleges have faced obstacles to offering
alternatives resembling those offered at some universities suggests a need for conversations that could yield greater alignment and transparency. Both the Common Core math standards and the alternative remedial sequences are now being implemented, however, which could make the time ideal to productively address alignment.

## The Algebra Assumption

Despite the ambiguity, at least for now, most math reforms in California and nationally assume the centrality of Algebra 2. If new K-12 math standards succeed in ensuring that more students master Algebra 2 content, differences over that emphasis may subside. But even in the best case scenario, it could take 10 years for new standards and teaching practices to be institutionalized and for students steeped in them to reach higher education. In the meantime, claims about the value of two years of algebra appear to go beyond what the research can bear.
"All students should take four years of mathematics in high school, at least through Algebra 2 or its equivalent, to be prepared for college and the workplace," according to Achieve, whose reports are often cited on the importance of Algebra 2. "The intellectual preparation that higher-level high school math courses provide maps closely to the kinds
of mathematical thinking that university educators believe are needed for success in college" (Achieve, 2008).

One problem with that assertion is that most of the related research is based on correlation: Since universities have generally required Algebra 2 either for admission or as a remedial course, it is by definition a powerful predictor of students' success because few students can progress in college without it. For STEM majors, roughly a quarter of fouryear university students, mastery of high school algebra unquestionably is correlated to college success. But there is little empirical basis for the claim that algebraic manipulation somehow makes all students more successful in their coursework.

Secondly, even if the research did establish a causal link between Algebra 2 and college success, it would not preclude the possibility that Statistics or other quantitative preparation could offer equivalent or even stronger preparation for students. In fact, a study by the National Center on Education and the Economy found that, besides middle school mathematics, most non-STEM en-try-level community college courses require grounding in areas that have rarely been taught in high school: modeling, statistics, and probability (NCEE, 2013). Though organizations such as Achieve commonly
refer to the value of "Algebra 2 or its equivalent," there has been little attention to defining what an equivalent might look like. And because of the prevailing assumption, there has also been very little research into how learning statistics or quantitative reasoning instead of algebra-based content might impact students' subsequent success.

Third, the ambiguity around the definition of Algebra 2 makes it hard to pinpoint precisely what content is deemed important. Some mathematical scientists are confident that, for general education preparation, a subset of Algebra 2 topics, rather than an entire course, is sufficient. As statistician Robert Gould of UCLA puts it, "Intermediate Algebra has many ideas and concepts and skills that are important for many disciplines. Whether they need to be packaged in what is now the Intermediate Algebra class is what I question." It may be that CSU math faculty, in paring back the content of the system's placement exam, are ahead of the curve.

Finally, the research linking academic success to algebra is typically based on high school course-taking (e.g., Adelman, 2006). But most colleges and universities use test scores-not course taking-to assess students' readiness for collegelevel math. So even if there were an empirical basis for the claim that al-
gebra prepares students better than any potential alternative, it plainly fails to validate the current system of testing students regardless of their prior course-taking. The final report in the Degrees of Freedom series will examine the role of placement policies in making remedial math the bottleneck it currently represents.

A case for alternative math sequences is beginning to emerge from the experiments at community colleges. Early evidence that the sequences lead to a marked improvement in students' passage of gatekeeper math courses is encouraging. Experiments at CSU campuses may add to this base of knowledge. If, over time, further studies show that the students do well in their subsequent studies and careers, there will be no empirical basis for opposing the alternatives. In order to develop an empirical understanding about the courses' effectiveness, however, it is important that transfer challenges don't thwart colleges from offering alternatives.

That could require a change in thinking at the university level, noted mathematician Bruce Cooperstein, who chairs the UC system's committee on preparatory education:

Over the years, we've taken the Rawlesian approach to students in math and science, the idea that potentially everybody might
want to become a neurosurgeon or a physicist, so everybody has to take lots of math. Colleagues in math and science are looking at it from the perspective of the students who end up in their classes, who want to be science majors and yet struggle with the mathematics in their courses. The physics professors are not coming into contact with students majoring in the digital arts or in psychology. If the role of math is as a gatekeeper, I'm totally opposed to it as a requirement.

## New Approaches to Math Articulation

Math requirements and their articulation across educational segments are in flux not just in California. The Carnegie Foundation's Statway program operates in eight states and Quantway, a quantitative reasoning sequence, in ten. Those were among the earliest alternative remedial programs and therefore have the most evidence to date. But several statelevel efforts are also being implemented, and each has tackled the articulation question to some degree:

- Texas community colleges are working with the New Mathways Project at the University of Texas at Austin to develop remedial pathways for Statistics and Quantitative Reasoning. In July 2014, through a negotiated rule-
making process, the state's higher education coordinating board approved a recommendation developed by representatives from institutions including UT-Austin, the Texas A\&M system, and a number of two-year colleges. Under the change, students may receive one of two designations based on completion of high school courses or college remedial courses: "ready for any freshmanlevel math course" or "ready for non-algebraically intensive mathematics courses." Students in the latter category who subsequently decide to pursue majors requiring College Algebra or Business Mathematics could be required to complete additional remedial courses or bridge courses.
- In 2013, when a task force in Georgia urged colleges not to require College Algebra for students outside of fields requiring Calculus, it explicitly said that for college-level courses in quantitative skills and mathematical modeling, new remedial prerequisites in addition to Intermediate Algebra should be developed.
- In Ohio, a task force concluded that math departments should remove College Algebra as the default mathematics course for non-STEM majors. It also recommended that colleges remove Intermediate Algebra as the sole
prerequisite for college math readiness and develop new prerequisite courses to ensure their transferability. A process is now under way to consider incorporating alternative prerequisites into the state's transfer module by Fall 2015.
- Colorado's community colleges are implementing a new quantitative literacy remedial sequence in addition to the standard algebraic literacy sequence. The goal is to prepare more students to transfer. There are three general education math courses accepted by the state's universities. The new sequence prepares students for either Liberal Arts Math or Statistics, which are accepted by majors such as history, English, sociology, and political science. Other majors, such as business, engineering, psychology, and economics, require algebra for transfer, so students would need to take the STEM remedial sequence.
- In Massachusetts, a 2013 task force on transforming developmental math recommended that colleges realign their developmental math courses with their college-level courses. While not a requirement, the task force's report indicated that it expects four-year institutions to accept revisions made by community colleges.

In each of these cases, faculty from universities and community colleges joined with policymakers to craft solutions with a goal of improving students' readiness for college in order ultimately to increase college completion rates.

## Looking Ahead

Shifting priorities for math preparation is a heavy lift in part due to math departments' penchant for the conventional pathway. Judy Kysh of San Francisco State University recalls:

For years, ever since I've been in math education, there has been discussion of the need to prepare students better, particularly students who will not go on and take Calculus, for Statistics, their statistical literacy, and their ability to use statistics in the many fields they do go into. At the same time, there has always been in math departments a strong demand to prepare students to go forward in the normal sequence through Pre-Calculus and Calculus. It's very hard to make a dent in that.

Many mathematicians are convinced of the centrality of algebra, and view a college experience without advanced algebra as somehow incomplete. To be sure, the idea of providing more academic exposure to more students is a compelling
one. But as non-math faculty and administrators increasingly enter the conversation, the new question being raised is whether that idea is compelling enough to dictate-or eliminate-opportunities for students such as Charles and Jodie, and whether it still makes sense to privilege algebraic learning over statistical competence. Elevated failure rates in courses such as Intermediate Algebra and College Algebra make it clear that the futures of hundreds of thousands of students could be at stake.

This could be a false choice unless the capacity of the postsecondary system grows to meet the demands of greater college attainment. Even under existing math requirements, not all students the community college system defines as "transfer ready" are transferring. That gap will likely only accelerate if the newly developed "associate degrees for transfer" swell the ranks of transferready students. If math requirements are broadened, even more students could potentially reach that benchmark.

Absent a change in the state's priorities, the state's universities would not have room for all of them. That concern has already arisen at UC Regents' and CSU Trustees' meetings in response to Governor Jerry Brown's interest in increasing the number of transfer students. "The
first conversation has to be that we have to fund higher education adequately, to ensure there is space for these students," noted Bill Jacob, a UC-Santa Barbara mathematician and former chair of UC's Academic Senate.

Under the status quo, then, math requirements are actually carrying the water for the state in restricting the number of transfer students it must pay for, effectively rationing access to higher education. When demand is too high, universities and departments seek filters to limit the number of qualified students, whether or not those filters are clear prerequisites. UCLA's psychology department, for example, requires undergraduates to pass physics, even though physics is rarely, if ever, used in any field of psychology.
"Unfortunately, many mathematics faculty accept the long tradition of their discipline as a filter and expect a large number of students to fail," wrote Bernard Madison, in his 2003 article. "This expectation casts a pall that hangs over many mathematics classrooms, causes additional students to fall, and increases resentment toward mathematics."

The filters mean that, completion goals notwithstanding, there is little room to expand opportunities for community college students, regardless of their achievements. Were the
requirements changed, making more students eligible to transfer, the public universities would face a choice between finding new criteria for limiting transfers or accepting more students than they have funding for (reducing their per-student funding rate). Similarly, if every high school student began earning A's in the required three-course math sequence, the universities might feel the need to begin requiring Pre-Calculus or Calculus as well (and, in fact, that is already happening at more selective campuses).

While the current admissions policies may seem overly restrictive to those seeking to improve community college student success, the universities' fiscal constraints, in effect, reinforce them. Such pragmatic considerations could make it hard to have a serious conversation in California about developing a coherent vision for math preparation. But the pragmatic needs of students could make such a conversation impossible to avoid.

As one community college math instructor at a 2014 student success conference noted,

In the old days, we could say we want an educated person to know math, but we never had to unpack that for our colleagues. One might have said an educated person needed to know Latin or
to have a class in philosophy, yet those have gone by the wayside. I think our professional community is at a crossroads right now in terms of how we answer this question of what is mathematics, what do students need, and how are we going to provide it. If we just provide the same answer we've always given, more people are going to ask why.

The time is ripe for these critical conversations. The adoption of new $\mathrm{K}-12$ math standards as well as community college experiments with alternative remedial sequences-not to mention recently approved pilots for community college four-year degrees-could provide an opening. Ideally, this conversation will look beyond mere alignment to focus on ensuring that math education in the 21 st century better serves students and the state.

To do this, California's education system should consider the following:

Intersegmental conversations are needed to deepen alignment across segments in math education. Rather than abstract discussions, these should be concrete projects, focused on meaningful alignment, such as those described below.

- Develop a stronger understanding of existing math offerings
by analyzing the range of quantitative reasoning requirements across segments and disciplines, and their implications for college readiness as well as prerequisite articulation for transfer.
- Analyze student progress in math across segments to understand roadblocks. Utilize the Common Core's dual emphasis on specific math topics as well as on standards of math practice as a framework to deepen the definition of mathematics learning.
- Include math faculty as well as faculty from client disciplines to develop shared priorities that are pragmatic, transparent, and supportive of student success.

Given the impressive results of innovations with alternative remedial sequences to date, the higher education segments should not adopt policies that would interfere with the ability to gather evidence on the effectiveness of these alternatives.

- Allow transfer of students in alternative remedial math sequences, at least on a pilot basis, in order to see how they perform at universities.
- Ensure that both UC and CSU research the outcomes of Statway and other alternatives, including students' performance in subse-
quent coursework. Both universities should articulate the principles under which such pathways are acceptable to provide clarity about the transferability of sequences other than Statway.
- Address the community college system's course coding issue, so that the outcomes of students in alternative math pathways can be studied over the long term.
- Explore development of bridge courses for students who take alternative math sequences and subsequently decide to pursue STEM majors. This includes addressing relevant policy changes.
- Research the real-life and career implications of statistics and quantitative reasoning training for students.

System leaders and state policy makers need to achieve greater clarity about the enrollment capacity constraints of the three higher education systems so that choices about entrance and graduation requirements can be separated from choices about enrollment capacity.

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## About Policy Analysis for California Education (PACE)

Policy Analysis for California Education is an independent, nonpartisan research center based at Stanford University, the University of California-Davis, and the University of Southern California. PACE seeks to define and sustain a long-term strategy for comprehensive policy reform and continuous improvement in performance at all levels of California's education system, from early childhood to postsecondary education and training. PACE bridges the gap between research and policy, working with scholars from California's leading universities and with state and local policymakers to increase the impact of academic research on educational policy in California.
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## About LearningWorks (LW)

LearningWorks aims to strengthen student achievement in community colleges. It does so by facilitating, disseminating and funding practi-tioner-informed recommendations for changes at the system and classroom levels, infusing these strategies with statewide and national insights. LearningWorks has been supported by the William and Flora Hewlett Foundation, the Walter S. Johnson Foundation and the James Irvine Foundation.

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[^0]:    ${ }^{1}$ PACE served as an intermediary organization and convener of cross-segmental conversations for the California Diploma Project.

