THE OPPORTUNITY EQUATION

Transforming Mathematics and Science Education for Citizenship and the Global Economy

Mathematics for Whom? The Top of High School Meets the Bottom of College

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2008

Prepared for the Carnegie Corporation of New York-Institute for Advanced Study Commission on Mathematics and Science Education

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All students deserve the chance to prepare for college. Furthermore, as children they deserve more than the chance; they deserve our determination to overcome any damage done to what they expect for themselves. They deserve pathways to college designed as preparation, not as obstacle courses more appropriate for selection than preparation. The essence of this paper is this: **the mathematics pathways to college must be analyzed, evaluated and redesigned using criteria grounded in the purpose of preparation, not selection.**

We want all students to graduate from high school eligible and ready for college. We need a high school mathematics course (or sequence of courses) that defines minimum eligibility for admission and placement in one or more credit bearing college mathematics courses. This course should be valuable to a student population with a broad variety of goals. This course should not be Algebra II.

Even if our purposes haven't changed, time alone justifies a fresh look at mathematics pathways to college.

Given the varied levels of preparation in mathematics students bring to secondary school, there should be multiple pathways to the common goal of college readiness. Some pathways should provide more time and instructional attention for students that need it to achieve college readiness in mathematics. Some, but not all, pathways should take science, technology, engineering, and mathematics (STEM) interested students toward and into Calculus. Other pathways should be designed for future humanities and social science majors that will teach the mathematics most useful to them.

States increasingly have adopted the posture, if not the policy, that equates high school graduation requirements with college eligibility requirements [EdTrust, American Diploma Project (ADP), and Achieve, Inc.]. This trend has not been the result of broad discussions with colleges about the appropriateness of using current eligibility requirements for this purpose. It comes from the good motives of policy makers to make college accessible for students historically underserved. Challenge our students and teachers and, like landing a man on the moon, they will respond. Passion for the excellence of the motive does not justify the leap to the 'moon shot' strategy for achieving it. This paper proposes a strategy that aims to help more students achieve success.

The motivation for equating college eligibility with high school graduation is extremely important for equity reasons. When the opportunity to take college preparatory courses is decided by schools, or counseled students, many who could and should go to college never get the chance. The K-12 system is notoriously unfair in its allocation of precisely this set of opportunities.

In the past, high schools viewed selecting students for college preparation as part of its job. Teachers and counselors identified students who were "college material" and programmed them into tracks of courses that prepared them for college. Of course, prevailing prejudices exerted their ugly influence with the result that many were unjustly excluded from the chance. The present shift away from premature selection and tracking is long overdue.

All the biases of the society extend into the internal social interplay of students and schools and are reflected in the pathways and courses taken. Simply by making college eligibility the default curriculum, many who otherwise would never have had the chance will get the chance and make good use of it. Certainly, this is worth a lot. But is it worth the cost in students who could have graduated from high school, but now won't? Is it worth the cost to good students from poor communities who formerly found an academic haven in a selective pathway through advanced mathematics courses but who, in a more inclusive system, are now swept up with other students in very different courses with the same names? We do not know the proportions of winners and losers in these trade-offs. Why don't we act deliberately to reduce the unfair costs and increase the overdue benefits?

We do not know if success in Algebra II, the default eye of the needle, is attainable by the whole population. It is a more technically demanding course than any before it, and less coherent and focused than any after it. Sure, better preparation in earlier grades, interventions to support students having difficulties, better prepared and supported teachers, better programs, and so on will all help if the funding and leadership show up. But something else will also help.

Let's make rules that serve our purpose rather than close our eyes and let grandfather policies and courses designed for a quite different purpose rule our grand children. I am a grandfather and these rules were made by my grandfather's generation. Even if our purposes haven't changed, time alone justifies a fresh look at mathematics pathways to college.

In mathematics, we inherit a course sequence for selected college prep students. The high school Algebra II course is identified almost universally as the prerequisite course for enrolling in a credit bearing college mathematics course. As a consequence, placement examinations for entering college freshman emphasize Algebra II content. The high rate at which entering freshman flunk these placement tests is well known as the latest "entering freshman needing remediation statistic." The role played by Algebra II has hardly been re-examined even though the nation has shifted policies toward college preparation for all.

To accomplish this shift in policy and practice, three kinds of changes are needed:

1. Assign nearly all students to courses in a college preparation sequence

- Increase the number of such courses
- Increase the number of teachers qualified to teach advanced courses

2. Intensify instruction for students who are behind along the course trajectory toward college. Provide them with:

- Help with homework, tutoring
- Interventions targeting content from earlier grade levels
- Ramp up courses that use extra time (double period, extended day, sum-

mer, etc.) to boost students who are behind up to the college prep sequence

3. Redesign the content and pedagogy inside college preparatory courses to focus on all students learning what is essential for success in college rather than just accept whatever colleges set long ago as eligibility and placement standards. This will require course content mapped backward from empirical evidence of college success broader than STEM majors.

A rigorous analysis and evaluation of the mathematics actually needed to succeed in courses taken by humanities majors, for example, and empirical data on real graduates (from both levels of school) should precede the consensus building processes. A consensus of opinions is not enough to overcome the inertia of the status quo.

It is true that the America Diploma Project (ADP) and the recent National Mathematics Panel have issued "new" formulations of college readiness in mathematics. The new formulations, especially the revised ADP (2009) are a welcome improvement on traditional Algebra II. However, all of the effort has been on the high school end; examination of admissions and placement policies of colleges is equally needed. A rigorous analysis and evaluation of the mathematics actually needed to succeed in courses taken by humanities majors, for example, and empirical data on real graduates (from both levels of school) should precede the consensus building processes. A consensus of opinions is not enough to overcome the inertia of the status quo.

What is needed to fully examine this critical question is a serious investment of time, and broader participation and commitment from colleges, not just mathematics and mathematics education departments, but the broader academic senates and the professional schools. The question is not just, what mathematics freshman need to take that will allow them to proceed to higher level mathematics courses. It is also, what mathematics do humanities majors need? Pre-law students? Pre-med? And so on.

Indeed, one might start with a serious look at what mathematics humanities students really need in order to meet the quantitative requirement for College Algebra courses. Such courses are often much easier than Algebra II and rarely assume much understanding of Algebra II. ACT found that half the students who scored 21 on ACT mathematics coming out of high school earned a "B" in College Algebra as college freshman. A score of 21 is associated with content well below the Algebra II level. While College Algebra does not deserve to be grandfathered into a redesigned system without examination, it is symptomatic of how colleges deal with the issue of non-STEM majors in their own glass houses.

The Curriculum Survey conducted by ACT asks faculty from secondary and post-secondary institutions what mathematics is most important. There is an important/critical discrepancy between secondary and college faculties:

> *…..survey results also revealed a discontinuity between secondary preparation and postsecondary expectations. High school mathematics teachers gave more advanced topics greater importance than did their postsecondary counterparts. In contrast, postsecondary and remedial-course mathematics instructors rated a rigorous understanding of fundamental underlying mathematics skills and processes as being more important than exposure to more*

advanced math topics. These results suggest that high school mathematics instruction concentrating on building up fundamental understanding and rigorous application of fundamental principles will likely better prepare students for college-level math than will instruction that covers many content topics less rigorously.[1]

College faculty are asking for a slower, more thoughtful pace through fundamental topics rather than a rush through more and more advanced topics that is the hallmark of Algebra II.

Why should high schools impose on their students a tougher requirement (Algebra II) than colleges impose on theirs (College Algebra for humanities majors)? Because they have no choice, if they are to get their students eligible for college. The onus is on the colleges to re-evaluate admissions and placement rules.

Is this a call to water down high school mathematics? No. We are asking all of today's students to pass Algebra II tests that most of the humanities faculties of even the best colleges, most education policy makers, most of our elected officials could not pass. Indeed, requiring Algebra II will lead, if it hasn't already, to a downward spectrum of courses with the same name, Algebra II, but with degrees of down-watering negotiated on the spot between equally frustrated teachers and students. Who is served well by this pretense?

What is wrong with Algebra II as the gateway course for all? More than any other high school course, it is technically demanding. In a short time, students must learn too many complicated and abstract notations, manipulation skills, exotic (to them) concepts, and topics whose coherence will not be apparent until several courses later. Unless a student goes on to Calculus and enters a field where it is used, he or she will probably never use the mathematics of Algebra II.

Comparison to high performing competitor nations provides a revealing perspective. In Singapore, for example, there is a fork in the road at age 16, 10th grade. The mathematics prior to this fork is common to over 75% of the population preparing for direct entry into university after upper secondary (age 19). The upper secondary course sequence for humanities, social science, and business majors (known as H1) differs from that for STEM majors (H2). Singapore has located some topics after the fork in the road so that H1 students do not study them—for example, complex numbers. Conversely, they have substituted a more substantial study of statistics earlier in the high school sequence so that H1 students do study more statistics than U.S. students.

A better U.S. course sequence would incorporate the most demanding technical content (e.g., extended work with logarithmic functions, the composition of functions, much of the manipulation of trigonometric functions, and complex numbers) into the next course (pre-calculus) designed for STEM-specializing students. In addition, a mathematics course that defines college eligibility is needed that prepares students for the non-STEM college majors and professional schools that would be better served by different mathematics (statistics?) or even less mathematics. Colleges need to define this course based on an evaluation of what is relevant for these majors and professional schools. Then the high schools can adapt.

We have a lot of good kids in this country who have lives that involve friendships, family, dating, music, sports, jobs, and growing up. They are ready to work hard at school, study what they are asked to study, and do their homework. They are ready to be good parents, citizens, and employees. A blind barrier like Algebra II is likely to push them out of high school before graduation under circumstances likely to turn them against a society that did not treat them fairly. We should not let this happen. Students deserve a better chance to learn advanced mathematics that will matter to them no matter what their major or work.

Our industrial competitors in Europe and Asia have, already in place and in further development, serious mathematics pathways through secondary schools for students who will major in non-scientific fields, or go directly into the workplace. We should look at what the competition is doing.

People in schools, of course, have not been blind to the current problems with Algebra II, nor are they likely to be in the future. They will figure out some way to make the rules realistic. Usually this leads to ad hoc watering down of Algebra II courses, hidden tracking, and other means of mitigating the perverse effects of well meant, but poorly designed policies. Another unwelcome side-effect is that many students who plan to go on to major in the sciences will be stuck with these watered down courses. This serves hardly anyone well.

The 21st century workplace will be more technical than ever before. But does that equate to advanced mathematics for blue-collar workers in technical industries? In spite of the valiant attempt by ADP to justify content from Algebra II for technical workers, a more serious effort is needed. Interviews and observations of such technical workers are needed that determine what mathematics is in their thinking as they perform their tasks. It is not the mathematics that mathematicians see in the task, but the mathematics the worker needs in order to perform the task. I don't need to be an aeronautical engineer to catch a Frisbee. There is a big difference in the mathematics education needed to evaluate a formula by substitution (common in the workplace) and that needed to manipulate a formula algebraically.

One technical worker described learning his work responding to meters and graphical displays as "…like learning to drive a car. You get the feel for how much to turn it [a dial] to get it [a graphical reading] away from the yellow line…like putting your foot on the brake without skidding…." There is a lot of advanced mathematics behind the screens and meters and dials, but not a lot in the reasoning and competent performance of the worker.

Our industrial competitors in Europe and Asia have, already in place and in further development, serious mathematics pathways through secondary schools for students who will major in non-scientific fields, or go directly into the workplace. We should look at what the competition is doing. After all, these are the very countries who are outperforming us in international mathematics performance.

Singapore, for example, has three main pathways after age 16 that account for most (about 90%) of the population. Each pathway leads to a defined and valuable postsecondary future. From each, a defined route to the university exists. The pathways take different numbers of years to reach university.

The Singapore secondary school system begins after age 12, roughly grade 7 in the U.S. As in the U.S. there are two levels of secondary school: four years of lower secondary from age 12 to 16, typically followed by three years of upper secondary leading to university admission at age 19. Note that this is one more year of secondary school before university than U.S. students take.

What is striking by comparison to U.S. systems is the degree of specialization in Singapore secondary schools. There are some specialized schools beginning at age 12. Most students go to “government schools” roughly like our comprehensive schools, but early specialization in vocational schools is available to small numbers at this age.

The most important specialization is within the government schools. Here, there are three mathematics options defined by examination syllabi that lead students to one of three examinations at age16, the O, N(A), and N(T). Course content is common in these three strands for the first two years, but the content gets increasingly differentiated by the exam a student is preparing for in years three and four.

Seventy-five percent take the "O level" exams which usually leads to what they call “Junior College” which compares to our grades 11-13. Here students will study in one of three mathematics sequences: the H1 (humanities/social science/business), H2 (STEM) or H3 (honors). After two to three years of junior college, students take the “A level” exams which qualify them for university.

Those who take N(A) go on to a polytechnic institute (three years to polytechnic diploma and option to go to university). Some who take the O levels will also opt for a polytechnic institute. Some students from junior college can follow a route to a polytechnic diploma as an alternative to qualifying for university through A level examinations.

Those who take N(T) go on to technical institute (for two years) and then on to work opportunities closely articulated with school, and they have the option to enter a polytechnic institute as well.

The mathematics in these varied pathways includes a core common with the pathways leading to university so that students can switch pathways in either direction. But beyond the core, the mathematics fits the students’ goals. Topics from calculus and linear algebra are treated from a user’s perspective and given a less technical treatment and, in upper secondary, there is more statistics than in the U.S.

The Singapore example illustrates how it is possible to design multiple pathways to college entrance while still serving more specialized interests in the student population.

By design, it takes some students [e.g., N(A) and N(T)] longer to enter college because they are preparing for a different goal: work rather than college. The diagonal pathways to university do not involve any remediation, but a defined program of course work that includes topics not covered yet. These diagonal routes to the university exist, but are not typically traveled. Most N(T) students go directly into jobs. It should be noted that Singapore is far from perfect. The Singapore Ministry has identified correlations between student demographics, especially home language, and achievement and consequential participation in the various pathways. Language minorities are more likely to be in the N(T) pathways. They are working on reducing the correlations; it will be interesting to see what they do.

Some students are not headed for college majors in mathematics and sciences and they deserve a serious mathematics pathway designed for them.

For the many students in the U.S. who decide late to go to college, the pathway is riddled with ambiguities and obstacles known as 'remedial mathematics.' These pathways take longer than the comparable diagonals in Singapore, too long for many who must work full time to support self and family.

Some students are not headed for college majors in mathematics and sciences and they deserve a serious mathematics pathway designed for them. Recognizing this difference among students raises the specter of "tracking". Anyone concerned about equity and social justice, as I am, is rightfully fearful of tracking. But fear of tracking can do its own damage if it scares us away from talking about and acting on differences that are real.

The evils of tracking stem mainly from two sources:

- Denying some students the opportunity to take courses that lead to opportunity
- Placing some students in pathways that lead nowhere.

These evils can be mitigated, if not avoided altogether, by creating multiple pathways that lead to college, rather than relying on a single pathway through Algebra II. Schools should not place students. Students should be informed of the consequences and allowed to accept whatever challenges take them where they want to go. Eliminate pathways that lead nowhere. Eliminate pathways that are hoaxes. Open all pathways to all students.

Recommendation:

In order for all students to graduate from high school eligible and ready for college, we should establish high school mathematics standards that:

- Define minimum eligibility for admission and placement in one or more credit bearing college mathematics courses; and
- Offer multiple pathways to college readiness: these pathways should be valu-

able to a student population with a broad variety of goals. Some pathways should provide more time and instructional attention for students that need it to achieve college readiness in mathematics. Some pathways should take STEM (science, technology, engineering, and mathematics) interested students toward and into calculus.

Convene a select group of public and private college Presidents to oversee workgroups broadly constituted who will:

- Analyze and evaluate the status of traffic through current mathematics pathways K-16, pathways used in high performing countries, and mathematics actually needed or valuable in the full array of majors and professional schools;
- Formulate a few well designed college mathematics courses for the full array of majors and professional schools; and
- Formulate pathways through high school mathematics that prepare students for college mathematics courses from 2 that are more relevant than Algebra II.

Meanwhile, since these recommendations will take time, we should make use of the revised ADP benchmarks (anticipated 2009) as a worthwhile improvement over the status quo. To do this, colleges and high schools need to agree on a high school course sequence (with syllabi) for college-going students that serves more than the calculus taking majors. The courses would reflect the improved focus of the revised ADP benchmarks and be accepted for admissions and placement by colleges.

[1]Aligning Postsecondary Expectations and High School Practice: The Gap Defined Policy Implications of the ACT National Curriculum Survey® Results, 2005–2006, ACT